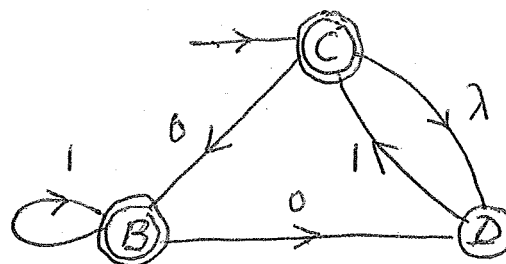


Answer all 6 questions. No calculators, formula sheets, or cellphones are allowed. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. BEGIN EACH OF THE 6 QUESTIONS ON A SEPARATE PAGE.

- (15) 1.(a) Define what is the language recognized by a DFA, $M = \langle Q, T, \delta, q_0, A \rangle$.
 (b) Let M be the NFA on the right. Find a DFA M_D which is equivalent to M .



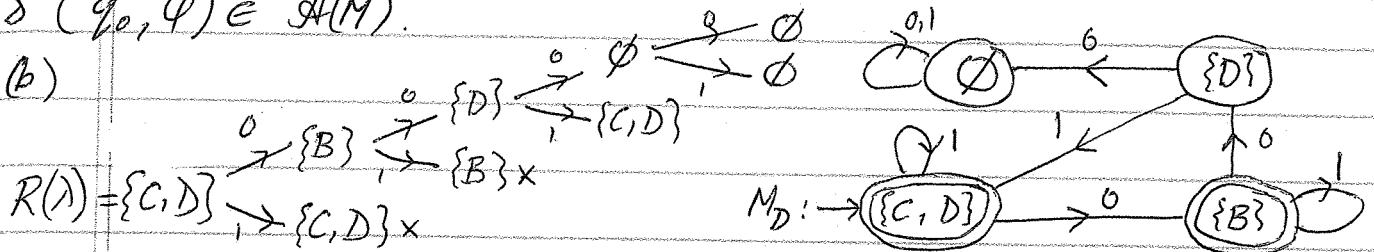
- (15) 2. Find regular expressions which describe the languages below.
 (a) $L_1 = \{\phi \in \{a,b\}^* : \phi \text{ contains both } bba \text{ and } ab \text{ as substrings}\}$
 (b) $L_2 = \{\phi \in \{0,1\}^* : \phi \text{ contains at most one occurrence of the string } 01\}$.
- (20) 3. (a) Define what it means for a state q to be inaccessible in a DFA M .
 (b) Partition the states of the DFA, M below into blocks of indistinguishable states and then find the equivalent reduced machine, M_R .

	<u>A</u>	B	$\rightarrow$ C	<u>D</u>	<u>E</u>	F	<u>G</u>
0	F	A	D	F	F	G	F
1	B	C	B	C	G	F	E

- (15) 4. (a) Let $f(v) = [2 \cdot n_b(v) - 3 \cdot n_a(v) - 1] \pmod{4}$. Find a DFA, M which accepts the language, $L_4 = \{\phi \in \{a,b\}^* : f(\phi) = 2\}$.
 (b) If $\phi = aaba$ find $f(\phi)$ & check that it agrees with your DFA with ϕ as input.
- (20) 5. (a) Find a context-free grammar G which generates the language $L_5 = \{a^k b^n : n \geq 2k + 3\} \cup \{b^k c^n : n \leq 3k + 2\}$.
 (b) Find derivations in G for each of the strings: (i) $a^1 b^6$ and (ii) $b^1 c^3$.
- (15) 6. Let A , B , and C be languages based on the alphabet $\{0,1\}$.
 (a) Is it always true that $(A \cap B) \cdot C^* \subseteq (A \cdot C^*) \cap (B \cdot C^*)$?
 (b) Is it always true that $(A \cdot C) - (B \cdot C) \subseteq (A - B) \cdot C$? (Justify your answers.)

1(a) The language recognized by the DFA M is defined by $L(M) = \{\varphi \in T^* : \delta^*(q_0, \varphi) \in A(M)\}$, i.e. $\varphi \in L(M) \iff \delta^*(q_0, \varphi) \in A(M)$.

(b)



2.(a) $\dots bba \dots ab \dots$, $\dots ab \dots bba \dots$, $\dots bba \dots ab \dots$, or $\dots abba \dots$

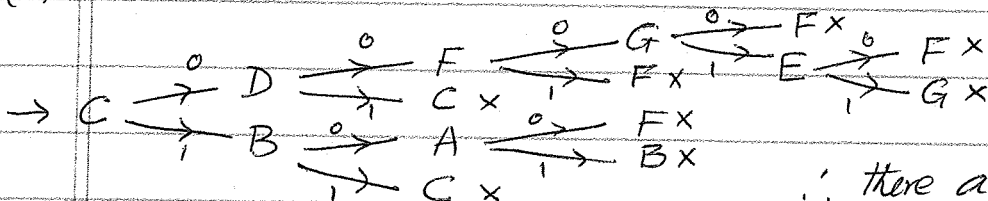
Reg. expr. for L_1 is $(a+b)^*(bba(a+b)^*ab + ab(a+b)^*bba + bbaab + abba)^*$.

(b) $11 \dots 100 \dots 0$ no 0's or $11 \dots 100 \dots 0.01.1 \dots 110 \dots 0 (a+b)^*$

Reg. expr. for L_2 is $1^*0^* + 1^*0^*.01.1^*0^*$

3(a) A state q in a DFA M is inaccessible if there is no string $\varphi \in T^*$ such that $\delta^*(q_0, \varphi) = q$.

(b) Let us check first for inaccessible states



$\therefore$ there are no inacc. states.

$P_0: \{B, C, F\} \{A, D, E, G\}$

$P_1: \{B, C, F\} \{A, D\}, \{E, G\}$

$P_2: \{B, C\}, \{F\} \{A, D\}, \{E, G\}$

$P_3: \{B, C\}, \{F\}, \{A, D\}, \{E, G\} = P_2$

$M_R: \rightarrow \{B, C\} \{F\} \{A, D\} \{E, G\}$

0 $\{A, D\} \{E, G\} \{F\} \{F\}$

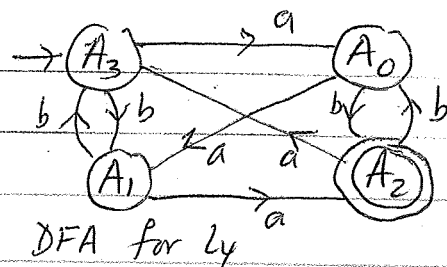
1 $\{B, C\} \{F\} \{B, C\} \{E, G\}$

4(a) Let A_i ($i=0,1,2,3$) keep track of the fact that $f(\varphi) \equiv i \pmod{4}$.

Then $f(\lambda) = 2n_b(\lambda) - 3n_a(\lambda) - 1 = 0 - 0 - 1 = -1 \equiv 3 \pmod{4}$

So A_3 will be the initial state of our DFA. Also A_2 will be the only accepting state because φ is in $L \iff f(\varphi) \equiv 2 \pmod{4}$.

4(a) ... Now $f(\varphi a) = [2n_b(\varphi a) - 3n_a(\varphi a) - 1] \pmod{4}$
 $= [2n_b(\varphi) - 3n_a(\varphi) - 1] - 3 \pmod{4} \equiv f(\varphi) + 1 \pmod{4}$
 And $f(\varphi b) = [2n_b(\varphi b) - 3n_a(\varphi b) - 1] \pmod{4}$
 $= [2n_b(\varphi) - 3n_a(\varphi) - 1] + 2 \pmod{4} = f(\varphi) + 2 \pmod{4}$



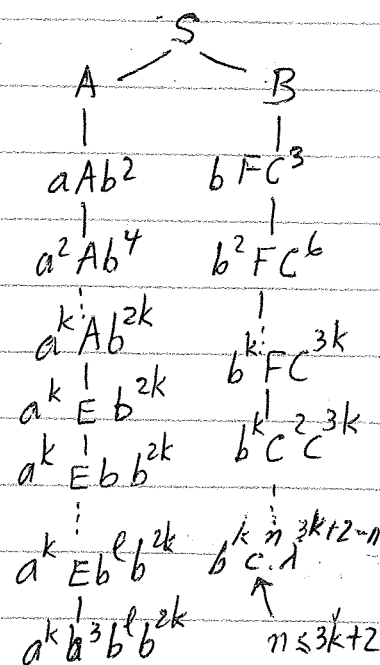
4(b) $f(aaba) = 2(1) - 3(3) - 1$ In our DFA: a a b a $\sqcup$
 $= 2 - 9 - 1 = -8 \equiv 0 \pmod{4}$. $\rightarrow A_3 \ A_0 \ A_1 \ A_3 \ A_0$

5(a) $\rightarrow S, S \rightarrow A|B$ (This gives the union)

$A \rightarrow aAbb|E, E \rightarrow Eb|bbb$ (gives $\{a^k b^n : n \geq 2k+3\}$)

$B \rightarrow bFCCC|CC, C \rightarrow c|\lambda$ (gives $\{b^k c^n : n \leq 3k+2\}$)

(b) $\rightarrow S \Rightarrow A \Rightarrow aAbb \Rightarrow aEbb \Rightarrow aEbbb \Rightarrow \overbrace{abbbbbb}^{a'b^6}$
 $\rightarrow S \Rightarrow B \Rightarrow bFCCC \Rightarrow bCCCCC \Rightarrow bcCCCC$
 $\Rightarrow bccCCC \Rightarrow bcccCC \Rightarrow \overbrace{bcccl\lambda}^{b'c^3}$



6(a). YES. Let $\varphi \in (A \cap B) \cdot C^*$. Then $\varphi = \alpha \cdot \gamma$ where $\alpha \in A \cap B$ and $\gamma \in C^*$. Since $\alpha \in A \cap B$, $\alpha \in A$ and $\alpha \in B$ and $\gamma \in C^*$. So $\varphi = \alpha \cdot \gamma \in A \cdot C^*$ bec. $\alpha \in A$ and $\gamma \in C^*$. Also $\varphi = \alpha \cdot \gamma \in B \cdot C^*$ because $\alpha \in B$ and $\gamma \in C^*$. Hence $\varphi \in A \cdot C^*$ and $\varphi \in B \cdot C^*$. So $\varphi \in (A \cdot C^*) \cap (B \cdot C^*)$. $\therefore (A \cap B) \cdot C^* \subseteq (A \cdot C^*) \cap (B \cdot C^*)$.

(b) YES. Let $\varphi \in A \cdot C - B \cdot C$. Then $\varphi \in A \cdot C$ and $\varphi \notin B \cdot C$. Since $\varphi \in A \cdot C$, $\varphi = \alpha \cdot \gamma$ where $\alpha \in A$ and $\gamma \in C$. Also since $\varphi \notin B \cdot C$, then $\alpha \cdot \gamma \notin B \cdot C$. Hence $\alpha \notin B$ (because if $\alpha \in B$, then since $\gamma \in C$, $\alpha \cdot \gamma \in B \cdot C$ - a contradiction). So $\alpha \in A$ and $\alpha \notin B$ and $\gamma \in C$. $\therefore \alpha \in (A - B)$ and $\gamma \in C$. Hence $\alpha \cdot \gamma \in (A - B) \cdot C$. So $\varphi \in (A - B) \cdot C$. $\therefore A \cdot C - B \cdot C \subseteq (A - B) \cdot C$.

END